Int. J. Heat Mass Transfer. Vol. 12, pp. 1595–1604. Pergamon Press 1969. Printed in Great Britain

CONVECTIVE HEAT TRANSFER FROM SMALL CYLINDERS TO MERCURY

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(Received 27 *September* 1968 *and in revised form* 7 *April* 1969*)*

Abstract—External heat transfer coefficients from short quartz-coated cylindrical hot-film anemometer probes in mercury were evaluated by subtracting the thermal impedance of the quartz coatings from the overall thermal impedance of the sensors. The results are compared with those by Sajben for long enamel-coated wires and with available theory for infinite cylinders, and correlation equations are given. The low sensitivity of finite cylinders may be qualitatively explained by the persistence of end losses, i.e. by two-dimensional conduction in the velocity-potential surfaces at arbitrary velocities. For comparison, results are also given for heat transfer to water.

NOMENCLATURE

a,	empirical parameter, equation (7);
b,	empirical parameter, equation (7);
C,	fouling factor, equation (6);
D,	outer diameter of sensor;
D_H,	diameter of platinum film or wire;
G,	Grashof number based on D and $T_H - T_\infty$;
G^*,	Grashof number based on D and $T_0 - T_\infty$;
k_f,	thermal conductivity of fluid;
k_Q,	thermal conductivity of quartz or enamel coating;
L,	length of sensor;
N,	Nusselt number based on $T_H - T_\infty$;
N_0	value of N at $R = 0$;
N^*,	Nusselt number based on $T_0 - T_\infty$;
N_0^*,	value of N^* at $R = 0$;
P,	Péclét number (σR);
R,	Reynolds number based on D and free-stream velocity;
T_H,	temperature of hot-film or hot-wire;
T_0,	temperature of surface of sensor;
T_∞,	temperature of undisturbed fluid;
Γ,	Euler's constant (0·5772...);
μ,	micron [10^{-6}m];
σ,	Prandtl number;
log( ),	natural (or Napierian) logarithm function;
O( ),	"Large oh" of Landau order notation, $\lim_{\varepsilon \to 0} \frac{1}{\varepsilon} O(\varepsilon) = \text{const.}$

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INTRODUCTION

THE DEVELOPMENT of cylindrical, quartz-coated, hot-film probes† has prompted a number of investigations into its characteristics for use as an anemometer in electrically conducting fluids such as mercury. In this use the fluid velocity is determined from the rate of heat loss from the cylinders, and so a knowledge of the heat transfer from cylinders to air, water, oils and transfer from cylinders to air, water, oils, and similar fluids has been much studied, but there is little similar information in low Prandtl number fluids such as liquid metals. In particular, information on external heat transfer coefficients is scarce.

Previous experiments performed in mercury with commercially available probes involved

† By Thermo-Systems, St. Paul, Minnesota.

cylindrical sensors having aspect ratios on the order of 20 [1–4]. An earlier study with long enamel-coated hot-wires having aspect ratios of about 100 had been made by Sajben [5, 6]. As we shall see, variation of the aspect ratio has considerable effect on the heat transfer.

In this paper we review empirical and theoretical formulae for steady convective heat transfer from infinite cylinders, suggest theoretical approaches for finite cylinders, and present new data on heat transfer rates from short cylinders to mercury and water. Although some natural convection data are reported here, we have been unable to find pertinent theory or experiments for comparison (short vertical cylinders, low G^*) and consequently restrict our review to forced convection.

INFINITE CYLINDERS—REVIEW

Empirical formulae

Here we list some empirical correlations to be used for comparison with our experimental results for water. In 1932 Ulsamer [7] correlated available data for air, water and several oils ($0{\cdot}7 < \sigma < 1000$) by the expressions

$$\left.\begin{aligned} N^* &= 0{\cdot}91\sigma^{0.31} R^{0.385}, \qquad 0{\cdot}1 < R < 50 \\ N^* &= 0{\cdot}60^{0.31} R^{0.50}, \qquad 50 < R < 10000 \end{aligned}\right\} \quad (1)$$

where N^* is the Nusselt number based on $T_0 - T_\infty$, T_0 is the temperature of the surface of the cylinder, T_∞ is the temperature of the undisturbed fluid, σ is the Prandtl number of the fluid, and R is the Reynolds number based on free-stream velocity and cylinder diameter.† Ulsamer's correlation was superseded by Kramers [8] with

$$N^* = 0{\cdot}42\sigma^{0.20} + 0{\cdot}57\sigma^{0.33} R^{0.50}, \quad 2 < N^* < 20$$

because he objected to the discontinuity in slope of equation (1) at $R = 50$. The objection may be unwarranted because the discontinuity may be indicative of the inception of vortex shedding. For wires in water Piret [9] obtained the correlation

$$N^* = 0{\cdot}965\sigma^{0.3} R^{0.28}, \qquad 0{\cdot}08 < R < 10, \quad 2{\cdot}37 < \sigma < 5{\cdot}64.$$

None of the foregoing correlations are expected to be applicable to liquid metals, whose Prandtl numbers seldom exceed 0·04. Sajben [5, 6], however, made careful measurements of heat transfer rates to mercury from enamel-coated wires having aspect ratios of about 100, and his data are compared with ours later.

Theoretical results

The theoretical approaches to the steady state convection problem have usually required that the structure of the velocity field be approximated by forms amenable to solution of the heat equation. The physical properties have usually been assumed constant and the temperature of the cylinder assumed uniform. Complications such as viscous dissipation and radiation have been ignored.

First we note the zero-flow solution $N^* \to 0$ as $R \to 0$; it is well known that no steady state can be reached for pure heat conduction from an infinite cylinder.

One class of theories uses an Oseen-type approximation of the heat equation in which the velocity field is approximated everywhere by the undisturbed free-stream velocity. The approximation is therefore valid only in the limit $P \to 0$ in the absence of natural convection currents. The result was given by Cole and Roshko [10] as

$$N^* = 2/[\log(8/P) - \Gamma] + O(P) \qquad (2)$$

where Γ is Euler's constant (0·5772...) and $P = \sigma R$ is the Péclét number. This was extended to $O(P^4)$ by Illingworth [11]. The experiments of Collis and Williams [12] show that equation (2)

† The fluid properties in (1) were evaluated as an integrated average over the temperature range T_0–T_∞, whereas those of the other investigations reported here were evaluated at a mean-film temperature $(T_0 + T_\infty)/2$. The procedure in [1] was to evaluate at T_∞ the viscosity used in the Grashof number G^*, the mass density, and the specific heat capacity; and to evaluate at $(T_H + T_\infty)/2$ the viscosity used in R and σ, and the thermal conductivity.

is moderately successful as an asymptote for small P.

A related class of approximations takes into account the higher order structure of the velocity field at low R. Piercy and Schmidt [13] suggested using the Oseen-flow approximation to the velocity field for arbitrary R in convection problems. Using this approach, Broer *et al.* [14] performed a numerical calculation of N^* at one value of R for air and water. Recently Kassoy [15] used the method of matched asymptotic expansions to obtain the first correction term to (2). Slightly different results were obtained independently by Hieber and Gebhart [16] and by Wood [17], of whom the latter used an iterative technique beginning with the Oseen solutions to the Navier–Stokes and heat equations.

Another class of theories uses a potential flow (inviscid) approximation of the velocity field. The approach developed by Boussinesq [18] was to use streamlines and velocity potential lines as coordinates (equivalent to a conformal mapping of the cylinder into a short plate) and then to neglect conduction along the streamlines for $P \gg 1$. The result is

$$N^* = \frac{4}{\pi}\left(\frac{2}{\pi}P\right)^{\frac{1}{2}} = 1{\cdot}016\sqrt{P}. \tag{3}$$

King [19] included the conduction along the streamlines but used improper boundary conditions. Aichi [20] corrected King's analysis and obtained (3) for $P \gg 1$ and a result similar to (2) for $P \ll 1$. Similarly, Piercy and Winny [21] obtained (3) for $P \gg 1$ and exactly (2) for $P \ll 1$. The entire range of P for potential flow was solved by Tomotika and Yosinobu [22]; their solution contains Piercy and Winny's as special cases. That N^* is given by (2) for small P for both potential and uniform flows is an interesting coincidence, as noted in Corrsin's review [23]. We note here that the empirical equation

$$N^* = \frac{1 + 2{\cdot}03P^2}{2P^{\frac{3}{2}} + \frac{1}{2}[\log(8/P) - \Gamma]} \tag{4}$$

gives (2) for $P \ll 1$, (3) for $P \gg 1$, and differs from Tomotika and Yosinobu's calculations by less than 3 per cent. Finally, Grosh and Cess [24] reasoned that for large R in liquid metals the thermal boundary layer should be so much thicker than the viscous boundary layer that the heat transfer should be almost completely insensitive to the details of the flow (including wake and separation) in the vicinity of the cylinder. They reported equation (3) for the case of uniform surface temperature and similar formulae for other boundary conditions.

Recently Dennis, Hudson and Smith [25] have evaluated N^* over the range $0{\cdot}01 < R < 40$ by solving the Navier–Stokes and heat equations numerically. They obtained good agreement with the experiments of Collis and Williams [12] in air.

FINITE CYLINDERS

The theory of finite cylinders introduces the further complexity of three-dimensional variation of the field quantities. The problem is simplified somewhat if the cylinder is represented by a long ellipsoid of revolution of major axis L and minor axis D with uniform surface temperature. The separable coordinates for this boundary and Helmholtz's equation are prolate spheroidal coordinates.† The solutions for Helmholtz's equation are spheroidal wave functions (or Legendre wave functions) and for Laplace's equation are associated Legendre functions.

For no fluid motion the solution of Laplace's equation outside the spheroid gives [10]

$$N_0^* = 2\Big/\left[\log\coth\left(\tfrac{1}{2}\coth^{-1}\frac{L}{D}\right)\right] \simeq 2/[\log(2L/D)] \tag{5}$$

for $L/D \gg 1$. This result is intermediate between

† For example, see [26] or [27].

the values $N^* \to 0$ as $R \to 0$ for a cylinder and $N_0^* = 2$ for a sphere, where N_0^* is the value of N^* at $R = 0$.

The Oseen-type approximation can be applied by transforming the heat equation to the Helmholtz equation, as for the cylinder. There is one more separation equation than for the cylinder problem, and the boundary condition of the transformed temperature is slightly more complicated. When the coefficients in the general solution have been evaluated (we have not done this), N^* can be calculated by taking $P \ll 1$.

The potential flow approximation can also be generalized to the three-dimensional case. The analog of Boussinesq's transformation would be to let one coordinate be the streamlines and the other two be arbitrary on the potential surfaces. If one of these is the hyperboloidal coordinate, the resulting mapping should be similar to that for the cylinder, except being of finite width. The analogous high P approximation, which neglects conduction along streamlines, still involves two-dimensional conduction in the potential surfaces, so the problem is more difficult than for the infinite cylinder, where only one dimensional conduction, along the potential lines, occurs at high P.

This picture suggests that conduction in the fluid is more efficient for short cylinders than long cylinders with a corresponding reduction in sensitivity to velocity changes. Crudely speaking, the finite cylinder should have large end losses that persist at all velocities.

So far this discussion has been restricted to constant T_0. A more realistic case would be to allow the local resistivity of the hot-film or hot-wire (and hence the local Joule heating) to assume the value corresponding to the temperature of the fluid at the surface. This would cause the nonuniform temperature distribution mentioned in [1], p. 51. The temperature on the spheroid would satisfy a nonhomogeneous Churchill condition; in spheroidal coordinates, however, the gradient term contains a variable scale factor for the hyperboloidal coordinate that destroys the orthogonality of the eigenfunctions.

THE EXPERIMENT

The equipment used in this study is described in detail by Hill [1] and will be but briefly summarized here. The cylindrical hot-film sensors are sketched in Figs. 1 and 2. The sensor is a quartz fiber covered with a thin film of platinum whose length is defined by a thicker plating of gold at either end. This sensor is then soldered across the tips of two rigid needles, the platinum and gold covered with a thin layer of quartz, and

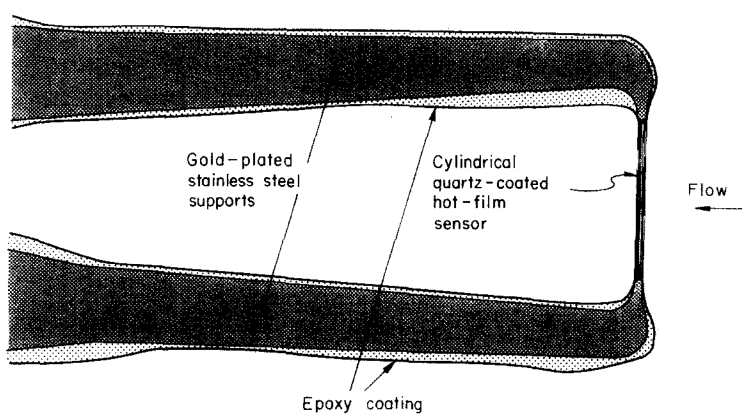

FIG. 1. Sensor and epoxy-coated supports of probe 8.

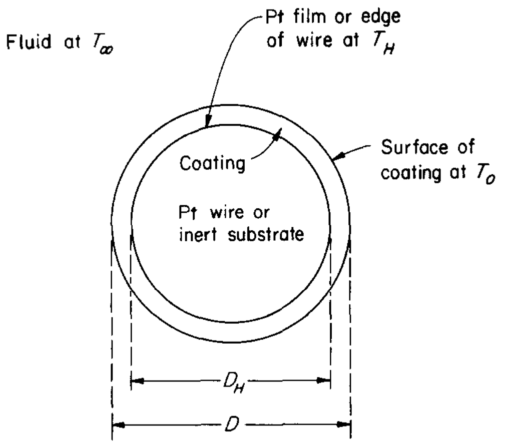

FIG. 2. Schematic of cross-section of sensor.

the remainder of the assembly insulated with a coating of epoxy resin. The diameter D, length L and aspect ratio L/D of the sensors were 53·6 μ, 1·06 mm and 19·8 for probe 7 and 57·9 μ, 1·03 mm and 17·8 for probe 8.

The probes were towed through a 4·88 m trough of mercury (or water) at speeds up to 56 cm/s. In the experiments reported here the sensors were vertical and always normal to the flow direction as indicated in Fig. 1. The probe rode on a cart beside the trough, and great pains were taken to avoid vibration and noise in the cart system, to achieve a uniform speed quickly, and to measure the speed with precision. The probes were heated with a Model 1010 constant-temperature anemometer circuit manufactured by Thermo-Systems. The resistance decks in the bridge of the circuit were calibrated to within 0·1 per cent, and the bridge was manually balanced during each run.

The fluid temperatures T_∞ were about 23°C with overheats $(T_H - T_\infty)$ from 35 to 40°C. The Prandtl numbers σ were about 0·0225 for mercury and 4·32 for water.

RESULTS

Model for calculating N^*

The heat transfer model used for calculating N^* is described in detail in [1] and [5] and is briefly summarized here. Let D_H be the diameter of the hot-film or hot-wire (Fig. 2) maintained at uniform temperature T_H. Heat is conducted through the uniform coating of quartz or enamel of thermal conductivity k_Q and outside diameter D and is convected away by the fluid, whose thermal conductivity is k_f and whose local external heat transfer coefficient is independent of azimuth. The overall Nusselt number N, based on $T_H - T_\infty$, can be measured and is related to N^* in this model by

$$1/N = 1/N^* + C, \qquad (6)$$

where C is a dimensionless fouling factor given by

$$C = \frac{1}{2}\frac{k_f}{k_Q}\log(D/D_H).$$

The assumption of azimuthal independence of the thermal impedance is crucial to the validity of the model. It is justified only by the uniformity of the quartz or enamel coating and by the tendency of the high k_f of mercury to smooth out variations in the local heat transfer coefficient.

In general C is quite difficult to determine. Sajben's approach was to shift the hot-wire calibration data along the $1/N$-axis until N^*, the adjusted value of N, coincides with the prediction of equation (2) at $P = 0{\cdot}05$. C is about 3·4 for his data shown here.

The approach used here is to calculate C directly after measuring the cross sections of the sensors viewed under a microscope. The ratio D/D_H was determined to be 1·16 for probe 7 and 1·12 for probe 8, corresponding to quartz thicknesses of 3·70 μ and 3·10 μ. From interpolated values of k_Q for vitreous quartz from Lange [28], C was calculated to be about 0·40 and 0·29 for probes 7 and 8 in mercury and about 0·028 for probe 7 in water. Because of the temperature dependence of k_Q, C depends slightly upon N. The temperature drop across the quartz layer varied from 13 to 37 per cent of the total overheat during operation in mercury, and from 2 to 15 per cent of the total in water.

Natural convection

The natural convection behavior of probes 7 and 8 in mercury is plotted in Figs. 3 and 4, in

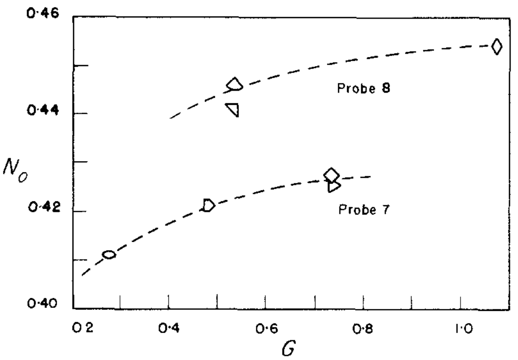

FIG. 3. Natural convection from vertical sensors in mercury; correlation based on $T_H - T_\infty$.

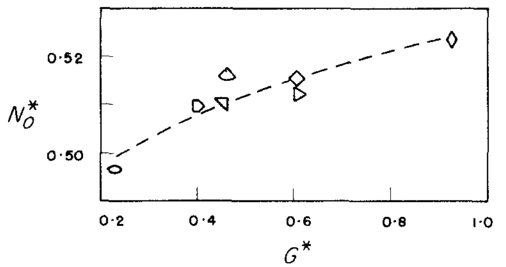

FIG. 4. Natural convection from vertical sensors in mercury; correlation based on $T_0 - T_\infty$.

which N_0 is the value of N at $R = 0$, and G and G^* are Grashof numbers based on $T_H - T_\infty$ and $T_0 - T_\infty$ respectively. The dashed line of Fig. 4 corresponds roughly to dlog N_0^*/dlog G^* = 0·03. The sensors were mounted vertically. Equation (5) estimates N_0^* as 0·543 and 0·560 for probes 7 and 8. Sajben's experimental value of 0·42 for N_0^* also corresponds reasonably close to the estimate 0·38 calculated with equation (5) for an aspect ratio of 100. Disagreement with (5) may arise from weak convection currents, from obstruction and distortion of the heat conduction path by the sensor supports, and from nonuniformity of T_0. The reasonable success of (5) and the fact that N_0^* is independent of sensor orientation ([1], p. 16) indicate that the natural convection currents in mercury are weak and that pure conduction is the dominant heat transfer mechanism.

For probe 7 in water at $G^* = 0·0114$, N_0^* was estimated from (6) to be 0·814, much larger than from (5). This indicates, together with the observed dependence of N_0^* on orientation, that the natural convection currents in water are rather strong.

That the geometry is important here is also evidenced by the fact that heat transfer rates calculated by the formulae of Eckert [29] and Ostrach [30] for vertical plates in mercury and water are an order of magnitude too low and overestimate dependence on Prandtl number.

Forced convection

Forced convection results for probes 7 and 8 in mercury are shown in Figs. 5 and 6. Unfortunately, there is no way of assessing the effects of distortion of the flow- and temperature-fields by the sensor supports (Fig. 1). The difference between the curves in Fig. 5 is attributable to the thermal impedances of the quartz coatings, as resolved by Fig. 6. Also plotted in Fig. 6 are some theoretical predictions for infinite cylinders† and the results of Sajben for long cylinders. The latter, except for $R < 2$ where three-dimensionality is important, fall away from Cole and Roshko's approximation towards the Boussinesq result and, in fact, fit Tomotika and Yosinobu's potential flow solution and the simplified formula (4) very well over the entire range. This behavior is in support of Grosh and Cess's conjecture mentioned earlier.

† The formulae of references [15] and [17], which take into account the higher order structure of the velocity field, are not applicable here because of divergences which occur for finite R, even though P is not large. The result of reference [16], however, does not diverge in this range because R does not appear in the formula for N^*. Calculations with this formula show a slight improvement over (2).

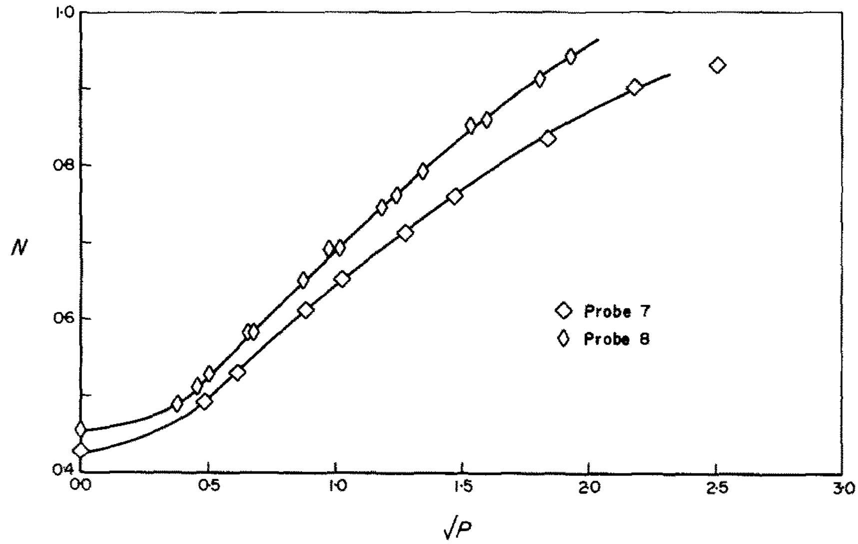

FIG. 5. Forced convection in mercury; correlation based on $T_H - T_\infty$.

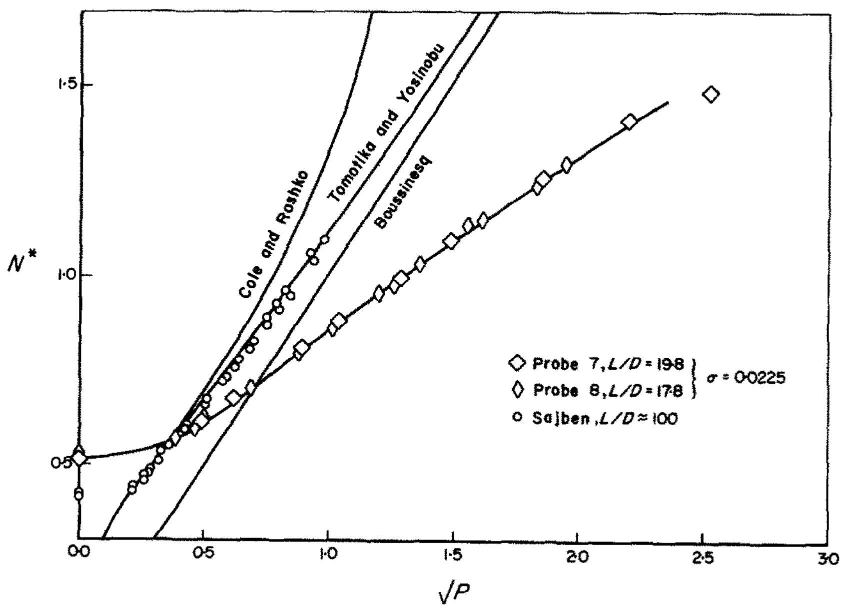

FIG. 6. Forced convection in mercury; correlation based on $T_0 - T_\infty$.

The short cylinders in mercury have noticeably lower sensitivity to velocity, in accordance with our previous suggestion concerning the persistence of two-dimensional conduction even in the large P approximation to the three-dimensional potential flow model. An independent verification of this low sensitivity is in the work of Malcolm [4] for a sensor with aspect ratio 16. His plots of $1/N_0 - 1/N = X$, which is relatively independent of C, showed sensitivity comparable to our plots of X for probes 7 and 8.

The data of Fig. 6 are correlated within about 2 per cent by the equation

$$N^* - a = [b^4 P^2 + (N_0^* - a)^4]^{\frac{1}{4}}, \qquad (7)$$

where $a = 0{\cdot}22 + 3{\cdot}2\ D/L$ and $b = 0{\cdot}95 - 9{\cdot}2\ D/L$. Equation (7) should provide a useful approximation for design purposes in the range $L/D > 15$ and $P < 3$. Also, N_0^* must be greater than a and may be estimated from (5) with a reduction in accuracy.

Forced convection results for probe 7 in water are shown in Figs. 7 and 8. The high value of N at $R = 49{\cdot}8$ is apparently due to the inception of vortex shedding ([1], p. 36). Because P was much larger for the water runs than for the mercury runs, the thermal boundary layer was much thinner, and the empirical formulae for long cylinders are expected to be valid here. The agreement in Fig. 8 with Ulsamer's correlation is only expected to be qualitative because of the doubtful validity of equation (6) for water. The slightly lower sensitivity may also be a manifestation of the end effect noted in mercury and of axial conduction in the quartz film. A similar comparison, in which C was neglected, has been made by James ([31], Figs. 13 and 14) for $L/D \simeq 20$ in water.

CONCLUSIONS

The heat transfer model of equation (6) has been fairly successful in the estimation of external heat transfer coefficients in mercury. For natural convection it was found that the convection currents are weak and that pure conduction is the dominant mode of heat transfer. For forced convection from short cylinders the sensitivity to velocity is much less

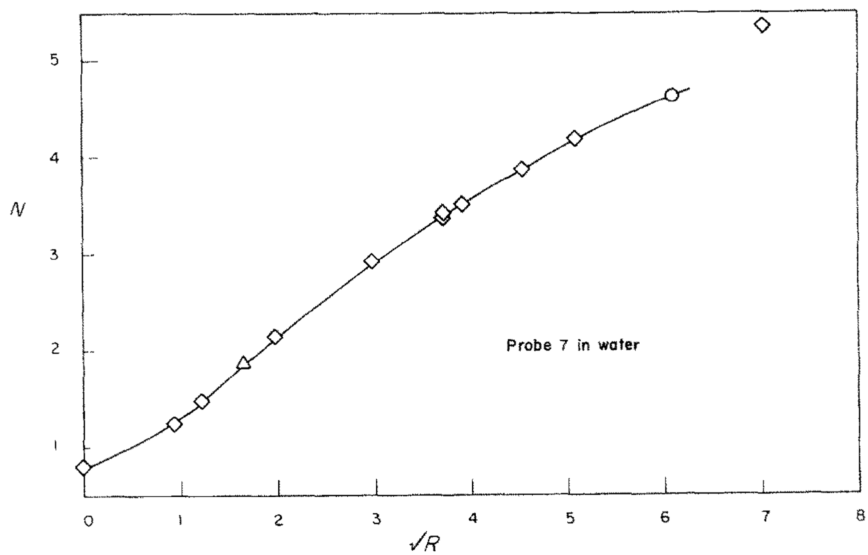

FIG. 7. Forced convection in water; correlation based on $T_H - T_\infty$.

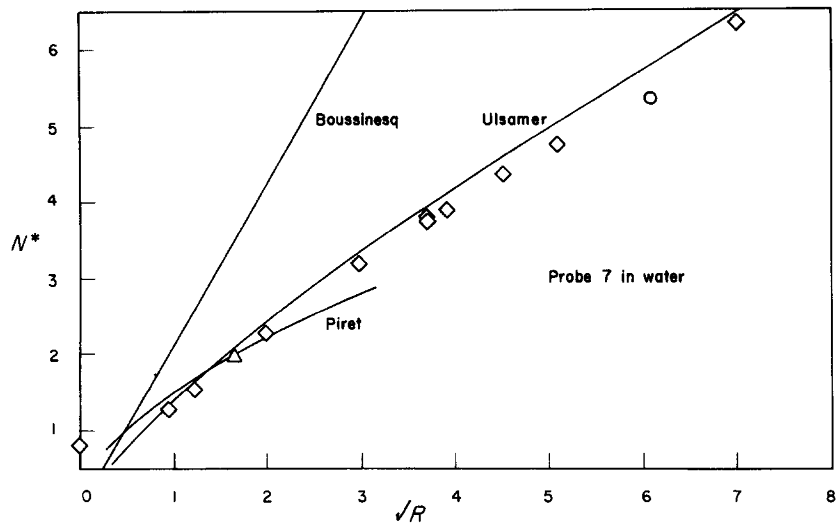

FIG. 8. Forced convection in water; correlation based on $T_0 - T_\infty$.

than for long cylinders, presumably because of conductive end loss effects in the fluid.

In water, for which equation (6) is less accurate, qualitative agreement with Ulsamer's correlation was obtained. The natural convection currents were much stronger than in mercury.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support of the National Science Foundation for this work. One of the authors (J.C.H.) was supported during part of this study by a NASA Space Sciences Traineeship.

REFERENCES

1. J. C. HILL, The directional sensitivity of a hot-film anemometer in mercury, Ph.D. Thesis, University of Washington, Seattle (1968).
2. D. C. EASLEY, Characteristics of a hot-film anemometer in a liquid mercury system, M.S. Thesis, Purdue University, Lafayette(1966).
3. M. HOFF, Hot-film anemometry techniques in liquid mercury, Grumman Research Department Memorandum RM-414 J. Grumman Aircraft Engineering Corporation, Bethpage (June 1968).
4. D. G. MALCOLM, Some aspects of turbulence measurement in liquid mercury using cylindrical quartz-insulated hot-film sensors, *J. Fluid Mech.* **37**, 701–714 (1969).
5. M. SAJBEN, Hot wire measurements in a liquid mercury jet subject to an axial magnetic field, Sc.D. Thesis, Massachusetts Institute of Technology, Cambridge (1964).
6. M. SAJBEN, Hot wire anemometry in liquid mercury, *Rev. Scient. Instrum.* **36**, 945–949 (1965).
7. J. ULSAMER, Die Wärmeabgabe eines Drahtes oder Rohres an einer senkrecht zur Achse strömenden Gas- oder Flüssigkeitsstrom, *Forsch. Geb. IngWes.* **3**, 94–98 (1932).
8. H. KRAMERS, Heat transfer from spheres to flowing media, *Physica, 's Grav.* **12**, 61–80 (1964).
9. E. L. PIRET, W. JAMES and M. STACY, Heat transmission from fine wires to water, *Ind. Engng Chem.* **39**, 1098–1103 (1947).
10. J. COLE and A. ROSHKO, Heat transfer from wires at Reynolds numbers in the Oseen range, *Proc. Heat Transf. Fluid Mech. Inst.*, pp. 13–25. University of California, Berkeley (1954).
11. C. R. ILLINGWORTH, *Laminar Boundary Layers*, edited by L. ROSENHEAD, p. 193. Clarendon Press, Oxford (1963).
12. D. C. COLLIS and M. J. WILLIAMS, Two-dimensional convection from heated wires at low Reynolds numbers, *J. Fluid Mech.* **6**, 357–384 (1959).
13. N. A. V. PIERCY and R. J. SCHMIDT, Use of Oseen's approximation in problems of heat transfer, *Phil. Mag.* **17** (7), 423–432 (1934).
14. L. J. F. BROER, C. J. HOOGENDOORN and A. KORTLEVEN, Some investigations on low-speed anemometry, *Appl. Scient. Res.* **A7**, 1–13 (1957).
15. D. R. KASSOY, Heat transfer from circular cylinders at low Reynolds numbers. I. Theory for variable property flow, *Physics Fluids* **10**, 938–946 (1967).
16. C. A. HIEBER and B. GEBHART, Low Reynolds number heat transfer from a circular cylinder, *J. Fluid Mech.* **32**, 21–28 (1968).
17. W. W. WOOD, Calculations for anemometry with fine hot-wires, *J. Fluid Mech.* **32**, 9–19 (1968).

18. M. J. BOUSSINESQ, Calcul du pouvoir refroidissant des courants fluides, *J. Math. Pures Appl.* **1** (6), 285–332 (1905).
19. L. V. KING, On the convection of heat from small cylinders in a stream of fluid: determination of the convection constants of small platinum wires with applications to hot-wire anemometry, *Phil. Trans. R. Soc.* **A 214**, 373–432 (1914).
20. K. AICHI, On the two dimensional convection of heat by the uniform current of stream, *Proc. Phys.-Math. Soc. Japan* **2** (3), 140–153 (1920).
21. N. A. V. PIERCY and H. F. WINNY, The convection of heat from isolated plates and cylinders in an inviscid stream, *Phil. Mag.* **16** (7), 390–408 (1933).
22. S. TOMOTIKA and H. YOSINOBU, On the convection of heat from cylinders immersed in a low-speed stream of incompressible fluid, *J. Math. Phys.* **36**, 112–120 (1957).
23. S. CORRSIN, in *Handbuch der Physik*, VIII/2 (Edited by S. FLÜGGE) p. 561. Springer, Berlin (1963).
24. R. J. GROSH and R. D. CESS, Heat transfer to fluids with low Prandtl numbers for flow across plates and cylinders of various cross section, *Trans. Am. Soc. Mech. Engrs.* **80**, 667–676 (1958).
25. S. C. R. DENNIS, J. D. HUDSON and N. SMITH, Steady laminar forced convection from a circular cylinder at low Reynolds numbers, *Physics Fluids* **11**, 933–940 (1968).
26. P. MOON and D. E. SPENCER, *Field Theory Handbook*, pp. 28, 201. Springer, Berlin (1961).
27. A. N. LOWAN, Spheroidal wave functions, *Handbook of Mathematical Functions*, edited by M. ABRAMOWITZ and I. A. STEGUN, Chapter 12. Nat. Bur. Stand., Washington (1965).
28. N. A. LANGE, *Handbook of Chemistry*, 9th edn, p. p. 1542. Handbook Publishers, Sandusky, Ohio (1956).
29. E. R. G. ECKERT and R. M. DRAKE, *Heat and Mass Transfer*, p. 315. McGraw-Hill, New York (1959).
30. S. OSTRACH, An analysis of laminar free-convection flow and heat transfer about a flat plate parallel to the direction of the generating body force. *NACA Tech. Note* 2635 (1952).
31. D. F. JAMES, Laminar flow of dilute polymer solutions around circular cylinders, Ph.D. Thesis, California Institute of Technology, Pasadena (1967).

TRANSPORT DE CHALEUR PAR CONVECTION DANS LE MERCURE A PARTIR DE PETITS CYLINDRES

Résumé—Les coefficients de transport de chaleur extérieur à partir de courtes sondes anémométriques cylindriques à film chaud revêtu de quartz vers du mercure ont été évalués en soustrayant l'impédance thermique des revêtements de quartz de l'impédance thermique, globale des capteurs. Les résultats sont comparés avec ceux de Sajben pour de longsfils émaillés et avec la thébrie disponible des longs cylindres et l'on a donné des équations de corrélation. La faible sensiblité des cylindres finis peut être expliquée qualitativement par la persistance des pertes aux extrémités, c'est-à-dire par la conduction bidimensionnelle dans les surfaces de potentiel de vitesse à des vitesses arbitraires. Dans un but de comparison, on a donné aussi les résultats pour le transport de chaleur dans l'eau.

Zusammenfassung—Äussere Wärmeübergangskoeffizienten an kurzen, quarzummantelten, zylindrischen Heiss-Film-Anemometersonden in Quecksilber wurden ausgewertet, indem vom gesamten thermischen Widerstand des Fühlers, der thermische Widerstand der Quarzschicht abgezogen wurde. Die Ergebnisse werden verglichen mit denen von Sajben für lange Drähte mit Emailmantel und mit der vorhandenen Theorie für unendliche Zylinder. Es werden Berechnungsgleichungen angegeben. Die niedrige Empfindlichkeit endlicher Zylinder kann qualitativ erklärt werden durch die Endverluste, dass heisst durch zweidimensionale Leitung in den Geschwindigkeitspotentialflächen bei beliebigen Geschwindigkeiten. Zum Vergleich werden Ergebnisse des Wärmeübergangs an Wasser angegeben.

Аннотация—Коэффициенты внешнего теплообмена между коротким цилиндрическим датчиком термоанемометра с кварцевым покрытием и ртутью, в которую он помещен, определялись путем вычитания термического сопротивления кварца из общего термического сопротивления датчика. Результаты сравнивались с данными Сейбена для длинных проволочек, а также с имеющимися теоретическими решениями для бесконечных цилиндров. Приведены коррелирующие уравнения. Низкую чувствительность коротких проволочек можно качественно объяснить существованием концевых потерь, т.е. двумерной проводимостью на поверхностях потенциального обтекания при произвольных скоростях. Для сравнения приводятся также аналогичные данные по теплообмену проволочек с водой.